**Assignment-2**

Q4:

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Answer:

Let's integrate the PDF over the valid range:

∫∫ Ke^(-x-y) dy dx

The range of integration for y is from 0 to x, and for x, it is from 0 to infinity.

∫[0 to ∞] ∫[0 to x] Ke^(-x-y) dy dx

Integrating the inner integral:

∫[0 to ∞] -Ke^(-x-y) evaluated from 0 to x dx

= ∫[0 to ∞] -K(e^(-x-x) - e^(-x-0)) dx

= ∫[0 to ∞] -K(e^(-2x) - e^(-x)) dx

Integrating the outer integral:

= ∫[0 to ∞] -K/2(e^(-2x) - e^(-x)) evaluated from 0 to ∞

= -K/2 (0 - 1) = K/2

For the PDF to integrate to 1, we must have K/2 = 1. Hence, the value of the constant K is 2.

Therefore, the constant K in the joint probability density function is 2.

**Q5:**

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The joint probability density function (PDF) of X and Y is given by Ke^(-x-y) for x >= y >= 0, and 0 otherwise. To find the conditional expectation, we can use the following formula:

E[X|Y=y] = ∫[0 to ∞] x \* f(X|Y=y) dx

where f(X|Y=y) is the conditional probability density function of X given Y=y.

Since the joint PDF is given by Ke^(-x-y) for x >= y >= 0, we can find the conditional PDF f(X|Y=y) by normalizing the joint PDF with respect to Y=y. We divide the joint PDF by the marginal PDF of Y evaluated at y:

f(X|Y=y) = (Ke^(-x-y)) / f(Y=y)

To find f(Y=y), we integrate the joint PDF over the range of X while fixing Y=y:

∫[y to ∞] Ke^(-x-y) dx = Ke^(-y) \* ∫[y to ∞] e^(-x) dx

= Ke^(-y) \* [-e^(-x)] evaluated from y to ∞

= Ke^(-y) \* (0 - (-e^(-y)))

= Ke^(-y) \* e^(-y)

= Ke^(-2y)

Now, we can substitute the conditional PDF f(X|Y=y) and compute the conditional expectation E[X|Y=y]:

E[X|Y=y] = ∫[0 to ∞] x \* ((Ke^(-x-y)) / (Ke^(-2y))) dx

= ∫[0 to ∞] x \* e^(-x+y) dx

= e^y \* ∫[0 to ∞] x \* e^(-x) dx

To evaluate this integral, we can use integration by parts or recognize it as the expectation of an exponential random variable with parameter 1. The expectation of an exponential random variable with parameter λ is 1/λ. In this case, λ = 1, so the expectation is 1/1 = 1.

Therefore, E[X|Y=y] = e^y \* 1 = e^y.

In conclusion, the conditional expectation E[X|Y=y] for the given joint probability density function is e^y.

**Q6**

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**Q8**

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Description automatically generated

An example of cyclostationary signal is the [linearly modulated digital signal](https://en.wikipedia.org/wiki/Modulation#Digital_modulation_methods) :

Q9:

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**Answer:**

To calculate the autocorrelation function R\_ss(t, t+τ) of the random process s(t) = A\*sin(2πft + θ), where θ is uniformly distributed in [0, 2π], we need to evaluate the correlation between the process at two different time instances t and t+τ.

The autocorrelation function is defined as the expected value of the product of two random variables at different time instances, i.e., R\_ss(t, t+τ) = E[s(t)s(t+τ)].

Let's calculate the autocorrelation function:

s(t) = Asin(2πft + θ)

s(t+τ) = Asin(2πf(t+τ) + θ)

R\_ss(t, t+τ) = E[s(t)s(t+τ)]

= E[Asin(2πft + θ) \* Asin(2πf(t+τ) + θ)]

= E[A^2 \* sin(2πft + θ) \* sin(2πf(t+τ) + θ)]

Since θ is uniformly distributed in the interval [0, 2π], it can be treated as a random phase uniformly distributed over the range.

Using the trigonometric identity sin(a)sin(b) = 1/2[cos(a-b) - cos(a+b)], we can simplify the autocorrelation function:

R\_ss(t, t+τ) = E[A^2 \* 1/2[cos(2πft + θ - 2πf(t+τ) - θ) - cos(2πft + θ + 2πf(t+τ) + θ)]]

= A^2 \* 1/2[E[cos(2πfτ)] - E[cos(4πft + 2πfτ)]]

The expected value of cos(2πfτ) can be calculated by integrating over the uniform distribution of θ:

E[cos(2πfτ)] = ∫[0 to 2π] cos(2πfτ) \* (1/(2π)) dθ

= 1/(2π) ∫[0 to 2π] cos(2πfτ) dθ

= 1/(2π) \* 0

= 0

The expected value of cos(4πft + 2πfτ) can also be calculated by integrating over the uniform distribution of θ:

E[cos(4πft + 2πfτ)] = ∫[0 to 2π] cos(4πft + 2πfτ) \* (1/(2π)) dθ

= 1/(2π) ∫[0 to 2π] cos(4πft + 2πfτ) dθ

= 1/(2π) \* 0

= 0

Therefore, both the terms involving cos(2πfτ) and cos(4πft + 2πfτ) in the expression for R\_ss(t, t+τ) have an expected value of 0.

As a result, we conclude that the autocorrelation function of the random process s(t) is zero for all values of τ. This indicates that the process s(t) is uncorrelated or orthogonal to itself at different time instances.

**Q10:**

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Description automatically generated with low confidence

**Mean/Expected value**

To find the expected value of u(t), which is denoted as E[u(t)], we can use the linearity of expectation and the fact that X and Y are independent standard normal random variables.

Given u(t) = Xcos(2πft) + Ysin(2πft), we can express the expected value as follows:

E[u(t)] = E[Xcos(2πft) + Ysin(2πft)]

Since X and Y are independent, we can calculate their expectations separately:

E[Xcos(2πft)] = E[X]E[cos(2πft)] = 0 \* E[cos(2πft)] = 0

E[Ysin(2πft)] = E[Y]E[sin(2πft)] = 0 \* E[sin(2πft)] = 0

Therefore, both terms involving X and Y in the expression for u(t) have an expected value of 0.

As a result, we conclude that the expected value of u(t), E[u(t)], is equal to 0.

**Auto-correlation**

To find the autocorrelation function r\_uu(t, t+τ) of the random process u(t) = Xcos(2πft) + Ysin(2πft), we need to calculate the correlation between the process at two different time instances t and t+τ.

The autocorrelation function r\_uu(t, t+τ) is defined as the expected value of the product of two random variables at different time instances, i.e., r\_uu(t, t+τ) = E[u(t)u(t+τ)].

Let's calculate the autocorrelation function:

u(t) = Xcos(2πft) + Ysin(2πft)

u(t+τ) = Xcos(2πf(t+τ)) + Ysin(2πf(t+τ))

r\_uu(t, t+τ) = E[u(t)u(t+τ)]

= E[(Xcos(2πft) + Ysin(2πft))(Xcos(2πf(t+τ)) + Ysin(2πf(t+τ)))]

Expanding the product and using the fact that X and Y are independent standard normal random variables, we get:

r\_uu(t, t+τ) = E[X^2cos(2πft)cos(2πf(t+τ))] + E[Y^2sin(2πft)sin(2πf(t+τ))]

= E[X^2cos(2πft)cos(2πfτ) + Y^2sin(2πft)sin(2πfτ)]

Since X and Y are independent standard normal random variables, they have a variance of 1. Therefore, E[X^2] = E[Y^2] = 1.

The autocorrelation function simplifies to:

r\_uu(t, t+τ) = cos(2πft)cos(2πfτ) + sin(2πft)sin(2πfτ)

= cos(2πfτ)

Thus, the autocorrelation function of the random process u(t) is given by r\_uu(t, t+τ) = cos(2πfτ). The value of the autocorrelation function depends only on the time difference τ and is independent of the specific time instance t.

**Is it wide-sense stationary?**

To determine if the random process u(t) = Xcos(2πft) + Ysin(2πft) is wide-sense stationary (WSS), we need to examine two properties: time-invariance and second-order stationarity.

Time-Invariance:

A random process is time-invariant if its statistical properties do not change with a shift in time. In this case, the random process u(t) is time-invariant because the sinusoidal functions, cos(2πft) and sin(2πft), are periodic and maintain their statistical properties over time.

Second-Order Stationarity:

A random process is second-order stationary if its mean and autocorrelation function do not depend on time. Let's examine these two properties:

a) Mean:

To determine if the mean of u(t) is time-invariant, we calculate the expected value:

E[u(t)] = E[Xcos(2πft) + Ysin(2πft)]

Since X and Y are independent standard normal random variables with zero mean, their expected values are both zero:

E[X] = E[Y] = 0

Therefore, the mean of u(t) is also zero, which does not depend on time.

b) Autocorrelation:

The autocorrelation function of a random process measures the correlation between the random variables at different time instances. To determine if the autocorrelation function of u(t) is time-invariant, we calculate the autocorrelation function:

R\_u(t1, t2) = E[u(t1)u\*(t2)]

For the given process, we have:

u(t1) = Xcos(2πft1) + Ysin(2πft1)

u(t2) = Xcos(2πft2) + Ysin(2πft2)

R\_u(t1, t2) = E[(Xcos(2πft1) + Ysin(2πft1))(Xcos(2πft2) + Ysin(2πft2))]

Since X and Y are independent, their cross-product terms will be zero:

E[XY] = E[X]E[Y] = 0

Therefore, the autocorrelation function simplifies to:

R\_u(t1, t2) = E[X^2cos(2πft1)cos(2πft2)] + E[Y^2sin(2πft1)sin(2πft2)]

Both cosines and sines are periodic functions, and their statistical properties do not change with time. Additionally, since X and Y are independent and have zero mean, their squares have constant variances.

Hence, the autocorrelation function R\_u(t1, t2) depends only on the time difference |t1 - t2|, and not on the specific time instances t1 and t2.

Based on the time-invariance of the mean and the constant autocorrelation function, we can conclude that the random process u(t) = Xcos(2πft) + Ysin(2πft) is indeed a wide-sense stationary (WSS) random process.